SOLUTION TO QUIZ 1

MATH 241

Differentiate $f(z) = \frac{x}{x^2+y^2} - \frac{y}{x^2+y^2}i$ as a complex function.

Proof. (Method 1) $f(z) = \frac{x-yi}{x^2+y^2} = \frac{\bar{z}}{|z|^2} = \frac{\bar{z}}{z\bar{z}} = \frac{1}{z}$. So $f'(z) = -\frac{1}{z^2}$.

(Method 2) Write $f(z) = u(x,y) + iv(x,y)$. So $f'(z) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}$, or equivalently, $\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}$. Now $u(x,y) = \frac{x}{x^2+y^2}$, $v(x,y) = -\frac{y}{x^2+y^2}$, so we can just calculate:

$$\frac{\partial u}{\partial x} = \frac{1 \cdot (x^2+y^2) - x \cdot 2x}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$\frac{\partial v}{\partial x} = -\frac{-y \cdot 2x}{(x^2+y^2)^2} = \frac{2xy}{(x^2+y^2)^2}$$

So $f'(z) = \frac{y^2-x^2}{(x^2+y^2)^2} + i\frac{2xy}{(x^2+y^2)^2}$.

Remark This is exactly of the same type of question as in Homework 2, section 17.5.21 & 22. If you did not do well on it, don't worry too much since you are allowed to drop two lowest quizzes, and all the future quizzes will be exactly of the same type of some question in homework. So just practice more on homework and digest what you learned from the lecture, feel free to come to my office hours to ask questions: DRL 3N2C, MW 4-5pm. Take it easy, do the homework and then have fun for the weekend! □